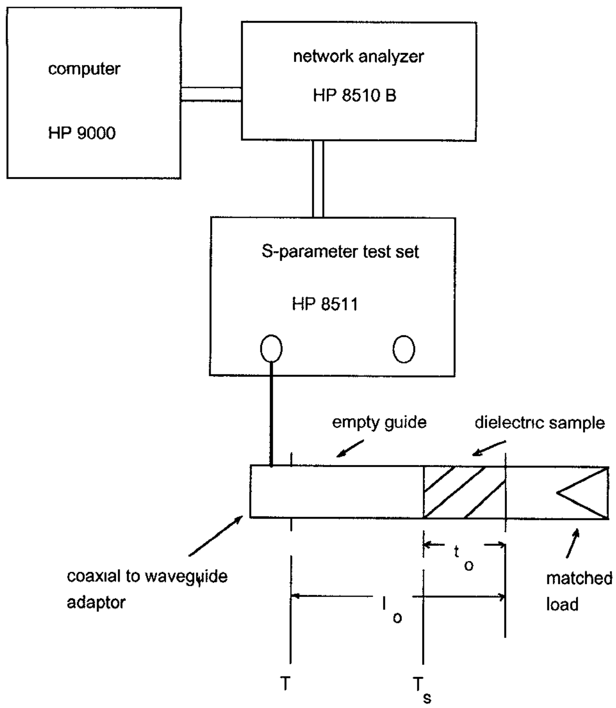

Fig. 2. HP 8510B automated network analyzer set-up for the measurement of the complex reflection coefficient of a dielectric sample embedded in a matched rectangular waveguide.

TABLE II
NUMBER OF ITERATIONS NEEDED BY A) MULLER' S AND B) DAVIDENKO' S METHODS TO FIND THE COMPLEX DIELECTRIC CONSTANT FROM REFLECTION COEFFICIENT MEASUREMENTS AND (12)

Guess	A	B
(3,-1)	7	15
(1,-3)	11	16
(4,-1)	7	15
(5,-4)	9	16
(2,-1)	7	15
(5,-1)	8	16
(1,3)	Fails	17
(5,4)	9	17
(1,6)	Fails	18
(2,10)	13	18

iterations required by each method to converge to the root within a specified tolerance (10^{-6}). Only Muller's and Davidenko's methods were compared. In this example, Muller's method appears to converge faster than Davidenko's for some initial guesses but diverges for other values. As in the previous example, the number of iterations required by Davidenko's method to converge is independent of the initial guess chosen ($N \simeq 17$). For the given measured data, both Muller's and Davidenko's methods yield a complex relative dielectric permittivity of $\epsilon_r \simeq (2.080465, -0.051842)$.

IV. CONCLUSIONS

In this paper we explored the capabilities of Davidenko's method as a complex root-search routine. It shows to be as promising as Muller's method and hence could be used as an alternative if Muller's method is slowly convergent or if it fails to converge to the root. The only apparent setback for Davidenko's method is that it requires the analytical expression of the first derivative (if it exists) of the complex function.

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Mutual Coupling Between Two Small Circular Apertures in a Conducting Screen

Sava V. Savov

Abstract—**With the use of the reaction integral and two-dimensional Fourier transform, an analytical expression for mutual coupling between two small circular apertures in a conducting screen, excited by normally incident plane electromagnetic wave, is obtained. Numerical examples for two different polarizations of the plane wave are investigated. The expression for the mutual admittance gives a correct value of the self admittance of a small aperture when the distance between the holes is equal to zero.**

I. INTRODUCTION

The problem of computing the mutual coupling between two equal apertures is a classical one. For the case of two narrow parallel rectangular apertures excited by a plane electromagnetic wave, it is dual of the problem of computing mutual coupling between two electric dipoles, which was solved for the first time by Carter [1] and later more accurately by King [2]. The problem of computing analytically the mutual coupling between two open circular waveguides was solved by Bailey [3]. The problem of the computing analytically the mutual coupling between two circular apertures in a conducting screen, excited by a plane wave, is studied in this paper.

Manuscript received July 16. 1990; revised April 28. 1992.
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IEEE Log Number 9204017.

0018–9480/93$03.00 © 1993 IEEE

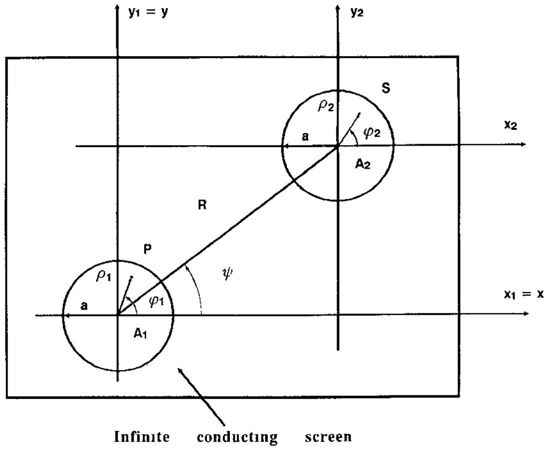

Fig. 1. The geometry of the problem.

II. Formulation and Solution of the Problem

The geometry of the problem is shown in Fig. 1. Two equal apertures A_1 and A_2 having radius $\boldsymbol{a}$ are considered. The first one has a center $O_1(0,0)$ and the second one $O_2(R\cos\psi, R\sin\psi)$, where R is the distance between two centers and ψ is specified in Fig. 1. For the sake of convenience two different polar coordinates (at each aperture) are chosen $(\rho_i, \varphi_i)(i = 1,2)$.

By definition, the mutual admittance between two apertures is given by the reaction integral [4]:

$$Y_{12} = -\frac{1}{V_1 V_2}\iint_{A_1} \overline{M}_1 \cdot \overline{H}_2^* dx_1 dy_1 \tag{1}$$

where according to the equivalence principle the magnetic current $\overline{M}_i$ is related to the electric field $\overline{E}_i$ by $\overline{M}_i = \hat{z} \times \overline{E}_i$($\hat{z}$ is unit vector along the z-axis). The magnetic field $\overline{H}_2$ is generated by the current $\overline{M}_2$ at the aperture A_1. The values $V_i(i = 1,2)$ are reference voltages at the apertures. Because of symmetry in the excitation $V_1 = V_2 = V_0$.

The electromagnetic field of the normally incident plane wave with a linear polarization is

$$\overline{E}_0 = \hat{x}E_0 e^{-jkz}, \overline{H}_0 = \hat{y}E_0/\eta e^{-jkz} \tag{2}$$

where $\eta = (\mu/\epsilon)^{1/2}$ is the intrinsic impedance of the medium and $\boldsymbol{k}$ is the wave number. It is convenient to set $E_0 = 1$. Since the equivalent magnetic current $\overline{M}_1 = \overline{0}$ outside the aperture A_1, the limits of the double integral in (1) can be extended to infinity. We take a two-dimensional transform over the current (and the magnetic field) at the plane of the screen ($z = 0$)

$$\tilde{\overline{M}}_1(k_x, k_y) = \iint_{-\infty}^{\infty} \overline{M}_1(x_1, y_1)e^{j(k_x x_1 + k_y y_1)}dx_1 dy_1. \tag{3}$$

With the use of Parseval's theorem, the expression for the mutual admittance (1) could be written in the spectral form

$$Y_{12} = -\frac{1}{V_0^2}\iint_{-\infty}^{\infty} \tilde{\overline{M}}_1 \cdot \tilde{\overline{H}}_2^* dk_x dk_y. \tag{4}$$

The magnetic field $\overline{H}_2$ of the current $\overline{M}_2$ can be determined by [6]:

$$\overline{H}_2 = -2\frac{j}{k\eta}[\nabla\nabla + k^2\overline{\overline{I}}]\iint_{A_2} G(r)\overline{M}_2 dx_2 dy_2 \tag{5}$$

where the scalar Green's function for the free space is

$$G(r) = \frac{e^{-jkr}}{4\pi r}$$

and $\boldsymbol{r}$ is the distance between a point $S(x_2, y_2)$ (source) and a point $P(x_1, y_1)$ (observer), $\overline{\overline{I}}$ is the unit dyadic and $\nabla = \hat{x}\partial/\partial x + \hat{y}\partial/\partial y$. The limits in (5) can be extended to infinity since $\overline{M}_2 = \overline{0}$ outside A_2, and the integral takes the convolution form. The application of the convolution theorem in (5) gives

$$\tilde{\overline{H}}_2 = -2\frac{j}{k\eta}\begin{bmatrix} k^2 - k_x^2 & -k_x k_y \\ -k_x k_y & k^2 - k_y^2 \end{bmatrix}\tilde{G}\tilde{\overline{M}}_2 \tag{6}$$

where the Fourier transform of the Green's function is [7]

$$\tilde{G} = -\frac{j}{2\sqrt{k^2 - k_x^2 - k_y^2}}$$

Now we are introducing the polar coordinates (β, α) in the spectral space $k_x = k\beta\cos\alpha, k_y = k\beta\sin\alpha$. Then (4) can be performed as follows:

$$Y_{12} = \frac{k^2}{V_0^2}\int_0^{\infty}\int_0^{2\pi} \tilde{\overline{M}}_1 \cdot \tilde{\overline{\overline{Y}}} \cdot \tilde{\overline{M}}_2^* \beta d\beta d\alpha \tag{7}$$

where $\tilde{\overline{\overline{Y}}}$ is the spectral admittance matrix given below:

$$\tilde{\overline{\overline{Y}}} = \frac{1}{\eta\sqrt{1-\beta^2}} \cdot \begin{bmatrix} 1 - \beta^2/2 - \beta^2/2\cos 2\alpha & -\beta^2/2\sin 2\alpha \\ -\beta^2/2\sin 2\alpha & 1 - \beta^2/2 + \beta^2/2\cos 2\alpha \end{bmatrix}$$

Now we must determine the Fourier transform of the magnetic currents $\overline{M}_i(i = 1,2)$. We can assume a uniform current $\overline{M}_i = \hat{y}M_0$ but a better approximation using edge conditions is [5]

$$\overline{M}_i = \hat{\rho}_i M_0 \sin\varphi_i\sqrt{1 - t_i^2} + \hat{\varphi}_i M_0 \cos\varphi_i 0.5\left[2\sqrt{1 - t_i^2} + \frac{t_i^2}{\sqrt{1 - t_i^2}}\right] \tag{8}$$

where $t_i = \rho_i/a$. It is convenient to introduce a global rectangular coordinates (x, y) instead of local polar coordinates $(\rho_i, \varphi_i)(i = 1,2)$. The exponent in the Fourier transform for the aperture A_2 is

$$k_x x_1 + k_y y_1 = k\beta R\cos(\psi - \alpha) + k\beta\rho_2\cos(\varphi_2 - \alpha) \tag{9}$$

The same expression holds for the aperture A_1 with $R = 0, (\rho_2, \varphi_2)$ replaced by (ρ_1, φ_1). Now we can find the Fourier transform for the magnetic current $\overline{M}_2$. By a substitution of (8) and (9) in (3) we have (10), which is shown at the bottom of the page, where $\phi = kR\beta\cos(\psi - \alpha), t = \rho_2/a$.

First, we take the integral over φ_2. Second, we replace sin $2\varphi_2$ and cos $2\varphi_2$ by the exponential functions. Third, we use the integral representation of a Bessel function of the first kind, order m [8]:

$$J_m(z) = j^{-m}/2\pi\int_x^{2\pi+x} e^{jm\varphi}e^{jz\cos\varphi}d\varphi \tag{11}$$

$$\tilde{M}x_2 = M_0 a^2/4e^{j\phi}\int_0^1\int_0^{2\pi}\left[\sqrt{1-t^2} - \frac{1}{\sqrt{1-t^2}}\right]\sin 2\varphi_2 e^{jkat\cos(\varphi_2 - \alpha)}tdtd\varphi_2$$

$$\tilde{M}y_2 = M_0 a^2/4e^{j\phi}\int_0^1\int_0^{2\pi}\left\{3\sqrt{1-t^2} + \frac{1}{\sqrt{1-t^2}} - \left[\sqrt{1-t^2} - \frac{1}{\sqrt{1-t^2}}\right]\cos 2\varphi_2\right\}e^{jkat\cos(\varphi_2 - \alpha)}tdtd\varphi_2 \tag{10}$$

where x is a constant. Fourth, we use the integral formula [9]:

$$\int_0^1 \frac{T_n(t)}{\sqrt{1-t^2}} J_\nu(\gamma t)dt = \pi/2 \frac{J\nu+n}{2}(\gamma/2)\frac{J\nu-n}{2}(\gamma/2) \quad (12)$$

where $T_n(t)$ is a Chebyshev polynomial, order n. After performing these operations we find

$$\begin{aligned} \tilde{M}x_2 &= M_0\pi a^2/2e^{j\phi}\sin 2\alpha K_2(\beta) \\ \tilde{M}y_2 &= M_0\pi a^2/2e^{j\phi}\{K_1(\beta) - \cos 2\alpha K_2(\beta)\} \end{aligned} \quad (13)$$

where ($\gamma = ka\beta$)

$$\begin{aligned} K_1(\beta) &= 1/\gamma^3[(3+\gamma^2)\sin\gamma - 3\gamma\cos\gamma] \\ K_2(\beta) &= 1/\gamma^3[(3-\gamma^2)\sin\gamma - 3\gamma\cos\gamma] \end{aligned} \quad (14)$$

Equation (13) defines the Fourier transform of the current $\overline{M}_2$. For the Fourier transform of the current $\overline{M}_1$ we can use the same expression after setting $\phi = 0$. Then we can find the mutual admittance by performing a double integration as given in (7). After taking integration over α and using (11) again, we have

$$Y_{12} = \frac{u^2}{8\eta}\int_0^\infty \{[(1-\beta^2/2)(K_1+K_2)^2 - 2K_1K_2]J_0(v\beta) - \cos 2\psi[\beta^2/2(K_1+K_2)^2 - 2K_1K_2]J_2(v\beta)\}\cdot\frac{\beta d\beta}{\sqrt{1-\beta^2}} \quad (15)$$

where $u = ka$ and $v = kR$.

The integral in (15) consists of two parts: 1) $\beta \in (0,1)$ where $(1-\beta^2)^{1/2}$ is real; 2) $\beta \in (1,\infty)$ where $(1-\beta^2)^{1/2} = -j(\beta^2-1)^{1/2}$ is imaginary. The first integral gives the real part and the second one—the imaginary part of the mutual admittance $Y_{12} = G_{12}+jB_{12}$. In the first integral since $\beta < 1$ and $u < 1$ (for small holes) we have $\gamma = u\beta \ll 1$ and with this approximation we get from (14) $K_1 = 2, K_2 = 0$. We find for the real part after performing the integration analytically

$$G_{12} = u^2/(8\eta)[L_1(v) - \cos 2\psi L_2(v)] \quad (16)$$

where

$$\begin{aligned} L_1(v) &= 1/v^3[(1+v^2)\sin v - v\cos v] \\ L_2(v) &= 1/v^3[(3-v^2)\sin v - 3v\cos v] \end{aligned} \quad (17)$$

For the imaginary part we have the exact expression

$$B_{12} = -u^2/(8\eta)[L_3(v) + \cos 2\psi L_4(v)] \quad (18)$$

where

$$\begin{aligned} L_3(v) &= \int_1^\infty [(\beta^2/2-1)(K_1+K_2)^2 + 2K_1K_2]J_0(v\beta)\frac{\beta d\beta}{\sqrt{\beta^2-1}} \\ L_4(v) &= \int_1^\infty [\beta^2/2(K_1+K_2)^2 - 2K_1K_2]J_2(v\beta)\frac{\beta d\beta}{\sqrt{\beta^2-1}} \end{aligned} \quad (19)$$

We must take the integrals (19) only numerically after setting $\beta = \cosh\tau$ and integration over τ at limits $(0,T)$ where T is a large enough number.

For the special case $v = 0 (R = 0)$ we can get expression for the self admittance $Y_{11} = G_{11} + jB_{11}$. From (17) for $v = 0$ we find $L_1 = 8/3, L_2 = 0$. In this approximation we obtain from (16)

$$G_{11} = \frac{u^2}{3\eta}. \quad (20)$$

We can take approximately $(\beta^2 - 1)^{1/2} \approx \beta$ (for $\beta \gg 1$) in the integral for B_{12} in (15). For $v = 0$ this procedure gives the approximate expression

$$B_{11} = -\int_0^\infty J_{3/2}^2(\gamma)\frac{d\gamma}{\gamma}. \quad (21)$$

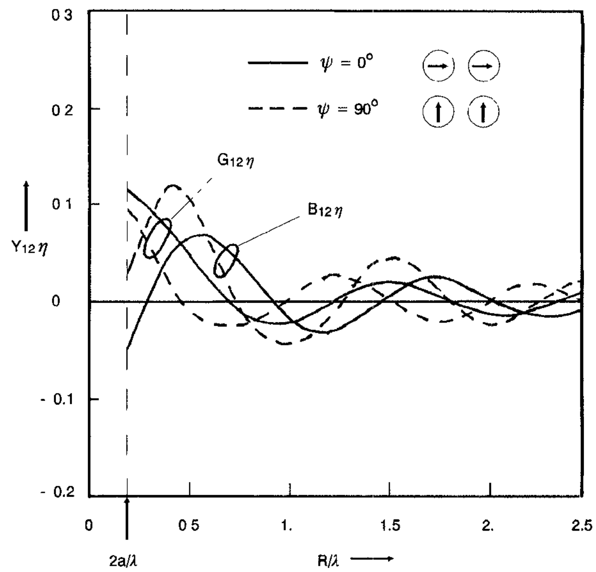

Fig. 2. Mutual admittance between circular apertures ($a/\lambda = 0.10$).

Now use the integral formula [9]:

$$\int_0^\infty J_\nu^2(\gamma)\frac{d\gamma}{\gamma} = \frac{1}{2\nu} \quad (22)$$

for the case $\nu = 3/2$ we get

$$B_{11} = -\frac{3\pi}{8\eta u}. \quad (23)$$

The expressions (20) and (23) for the self admittance completely coincide with the corresponding expressions, derived by Harrington [10] by a different approach. For the spacing $0 < R < 2a (0 < v < 2u)$ the formulas (16) and (18) are not valid since it is a nonphysical situation.

III. Numerical Results

Two numerical examples with radius $a = 3$ mm and $a = 4.5$ mm are evaluated for $\lambda = 30$ mm. G_{11} and B_{11} are computed by (20) and (23); G_{12} and B_{12} by formulas (16) and (18). (19) was computed by numerical integration using an adaptive Newton-Cotes formula [11]. The results for a normalized admittance $Y_{12}\eta$ as function of a normalized distance R/λ for the case $a = 3$ mm are shown in Fig. 2 (corresponding to $\psi = 0°$ and $\psi = 90°$) and for the case $a = 4.5$ mm in Fig. 3 (corresponding to the same angles). In the first case we obtain $Y_{11}\eta = 0.132 - j1.875$ and in the second one $Y_{11}\eta = 0.296 - j1.250$ (for both angles). The directions of the electric fields are shown by pointers. The numerical experiments showed that an accurate enough value for B_{12} is obtained even when the parameter T is relatively small ($T = 10$).

IV. Conclusion

In this paper an approximate analytical treatment for the complex mutual admittance $Y_{12} = G_{12} + jB_{12}$ between two small circular apertures in a conducting screen, excited by normally incident plane wave, is given. For the real part one can use the expression (16) and for the imaginary part—the expression (18). For the special case when the distance between the holes $R = 0$ we find the proper values for the self admittance $Y_{11} = G_{11} + jB_{11}$ in (20) and (23), which completely coincide with the results obtained by Harrington [10] by a different technique. The method proposed in this paper can be extended for the case of oblique incident plane wave.

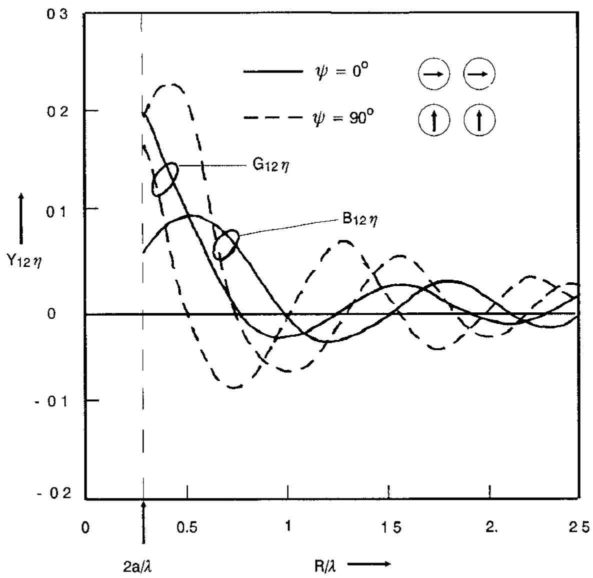

Fig. 3. Mutual admittance between circular apertures ($a/\lambda = 0.15$).

The canonical problem for the evaluation of the mutual admittance between two circular apertures which is solved here, can be used for the analysis of aperture antenna arrays, excited by waveguides and cavities [12], [13] and of frequency selective surfaces, excited by plane wave as well [14]. By using the duality we can find also the mutual impedance between two small circular conducting discs, excited by a plane wave.

ACKNOWLEDGMENT

The author is pleased to acknowledge helpful discussions with Prof. H. D. Hristov from the same university.

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Capacitance and Inductance Matrices of Coupled Lines From Modal Powers

Smain Amari

Abstract—**The capacitance and inductance matrices of a system of coupled lines are calculated from the modal powers. Knowledge of the propagation constants of the different modes, the eigencurrent matrix $[M_I]$ and the modal powers uniquely specify the two matrices. The present approach is tested both analytically and numerically.**

I. INTRODUCTION

Systems of coupled lines constitute an important part of modern integrated circuits. In many applications, the time response of this systems is necessary in describing the propagation of waveforms. Unfortunately, most available methods of analysis yield frequency dependent parameters with a complex frequency dependence. In principle, the time response could be obtained by taking an appropriate inverse integral transform such as Laplace or Fourier. Such an approach is, however, of limited practical value especially for a large number of lines. A method which seems to have found a large acceptance is based on devising equivalent lumped circuits for the system. Such circuits can then be designed using traditional methods of filter synthesis.

In this study, a method based on modal powers is used in determining the capacitance and inductance matrix of N coupled lines. Tripathi and Lee [1] give expressions for the capacitance and inductance matrices in terms of the characteristic impedances of the individual lines. Besides the fact that this assumes the possibility of having an accurate definition of these characteristic impedances, one needs to calculate these $N \times N$ parameters. It is shown here that it is possible to compute the two matrices without making use of the characteristic impedances whose privileged role is shifted to the normal modes and their powers. The use of modal powers which are uniquely defined from the electromagnetic field is expected to improve the accuracy of analysis of time response. This, however, is not rigorously justified in this paper. It also reduces the computational effort since one needs only N independent quantities, the modal powers, instead of the $N \times N$ characteristic impedances of the lines. Wei *et al.* and Weeks determined the capacitance matrix by solving the static boundary-value problem [2], [3]. This does not take into account the dispersive character of systems of coupled microstrips and the like.

Manuscript received December 26, 1992; revised April 17, 1992.
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IEEE Log Number 9204018.

0018-9480/93$03.00 © 1993 IEEE